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# Want to improve your cascade 2-stage AO system? Turn it into a high-performance woofer-tweeter!

<sup>1</sup> Henri-François Raynaud,<sup>1</sup>, Caroline Kulcsár, Nelly Cerpa-Urra<sup>2</sup>, Markus Kasper<sup>2</sup>

<sup>1</sup>Université Paris-Saclay, Institut d'Optique Graduate School, CNRS, Laboratoire Charles Fabry, 91127, Palaiseau, France

<sup>2</sup>European Southern Observatory, Karl-Schwarzschild Str. 2, Garching bei Munchen, Germany, 85748  
corresponding author: henri-francois.raynaud@institutoptique.fr

## Abstract:

2-stage cascade AO (CAO) systems enable to improve wavefront correction for high-contrast exoplanet imaging. However, 1<sup>st</sup> stage residuals entering the 2<sup>nd</sup> stage exhibit see-saw high-frequency oscillations. We propose a compensation procedure which turns the CAO into a woofer-tweeter configuration, thus significantly improving overall performance.

**OCIS codes:** (010.1080) Active or adaptive Optics; (010.1330) Atmospheric turbulence

## 1. 2-stage XAO architecture for improved exoplanet imaging

High-contrast imaging (HCI) with adaptive optics (AO) provides direct images of extra-solar planets. For optical and near-IR HCI, the AO-corrected residual halo stellar flux is the main source of noise. In order to obtain great contrast sensitivity for exoplanet imaging at small angular separations, it is therefore crucial to minimize this residual halo mainly due to AO loop delay. A straightforward approach to reduce this temporal delay would be to run these AO systems faster. A candidate architecture for future ultra-fast Extreme-AO (XAO) systems for high-contrast imaging is a cascade adaptive optics (CAO) system with two stages [1]. In a 2-stage CAO system a 1<sup>st</sup> stage running at a sampling frequency  $F_1$  corrects for the incoming turbulent phase, producing a residual phase that enters a faster 2<sup>nd</sup> stage running at a sampling frequency  $F_2 = n_{os}F_1$ , where  $n_{os}$  is the CAO oversampling factor.

The residual entering the 2<sup>nd</sup> stage is the sum of the oversampled version of the 1<sup>st</sup> stage residual  $\bar{\phi}^{res,1}$  and of the inter-sampling fluctuations of the incoming turbulence  $\phi$ , *i.e.* its averages over successive 1<sup>st</sup> stage sampling periods  $1/F_1$ . This correction plus upsampling procedure generates a 2<sup>nd</sup> stage input signal which exhibits a distinctive pattern of high-frequency see-saw fluctuations, even when the original incoming disturbance  $\phi$  is a low-frequency signal. Figure 1-Left shows these signals for a 2-stage system with  $F_1 = 1$  kHz and  $F_2 = 4$  kHz (so that  $n_{os} = 4$ ), when  $\phi$  is a sinusoidal signal at  $f_0 = 40$  Hz and where the two stages use standard integral action controllers. Even in this simple case, these high-frequency see-saw fluctuations are visibly not properly compensated by the 2<sup>nd</sup> stage controller. A more detailed time- and frequency-domain analysis of this partial compensation with oversampling issue is presented in [2]. As it turns out, compensating this quasi-cyclical 1<sup>st</sup> stage residual with a linear time-invariant controller is very difficult. (Indeed, its temporal statistics are not time-invariant, so that it cannot be accurately modeled as a stationary process.)

## 2. The woofer-tweeter trick

A simple way to circumvent this problem is to turn the 2-stage system into an equivalent woofer-tweeter system, where an optimized control effort is computed from the higher-rate measurements provided by the 2<sup>nd</sup> stage WFS, then split between the 2 DMs. To explain how to pull off this trick, consider any linear 2<sup>nd</sup> controller implemented in standard state-space form as

$$x_{k+1} = A_u x_k + B_u y_{2,k}, \quad (1)$$

$$u_{2,k} = C_u x_{k+1}, \quad (2)$$

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where  $u_{2,k}$  and  $y_{2,k}$  are the 2<sup>nd</sup> stage control and WFS measurement. Assuming for the sake of simplicity that the 2<sup>nd</sup> stage operates in a linear regime and that the computational delays for both AO loops is one frame,  $y_2$  is a function of the residual wavefront  $\phi^{\text{res},2}$  which depends on corrections performed by the 2 stages, namely

$$\phi_k^{\text{res},2} = \phi_k + N_2 u_{2,k-1} + N_1 u_{1,k-n_{\text{os}}} \quad (3)$$

where  $u_{1,k}$  is the oversampled 1<sup>st</sup> stage control action and  $N_1$  and  $N_2$  are the two DMs influence functions. In a configuration where the 2<sup>nd</sup> stage would operate alone (*i.e.*, with  $u_1 = 0$ ), equations (1-2) would remain identical, while the residual wavefront would become  $\phi_k^{\text{res},2} = \phi_k + N_2 u_{2,k-1}$ . Thus, to make the cascade 2<sup>nd</sup> stage control exactly equivalent to this stand-alone version, it suffices to make these two different expressions of  $\phi_k^{\text{res},2}$  equal, and thus to replace (2) by  $u_{2,k} = C_u x_{k+1} - N_2^\dagger N_1 u_{1,k+1-n_{\text{os}}}$ . This way, the control effort  $C_u x$  computed by the 2<sup>nd</sup> stage controller is thus split between the 1<sup>st</sup> and 2<sup>nd</sup> stage DMs.

This woofer-tweeter trick was applied to the 2-stage configuration in [2], using the simulation package OOMAO in MATLAB [3]. The two controllers use a truncated basis of Karhunen-Loève (KL) modes. The two stages run at  $F_1 = 1$  kHz and  $F_2 = 4$  kHz. Figure 1-Right compares the resulting residual modal phase variances for the 1<sup>st</sup> stage integrator operating alone (green curve), a conventional 2<sup>nd</sup> stage integrator (blue curve), the same 2<sup>nd</sup> stage integrator implementing the woofer-tweeter trick (red dotted line) and a 2<sup>nd</sup> stage LQG controller based on a modal turbulence model and also using the woofer-tweeter trick (red curve). The corresponding Strehl ratios (in I band) estimated from the residual variances are respectively 51.7 %, 62.9 %, 66.1 % and 66.5 %. Thus, implementing the woofer-tweeter trick with a standard integrator increases the SR by more than 3 points. An LQG predictive control improves performance further, especially for low-order modes – which should result in a significant improvement in terms of HCI performance.

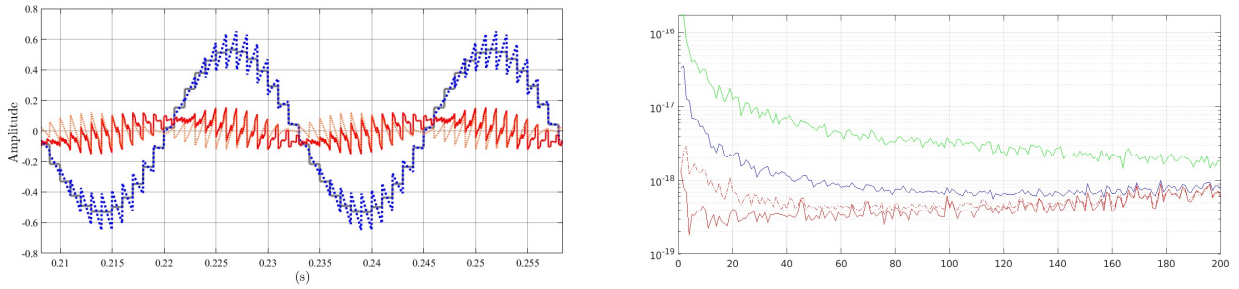


Fig. 1. *Left*: For an input sinusoidal signal of amplitude 1 at  $f_0 = 40$  Hz: inter-sampling signal  $\phi^{\text{inter}}$  (dotted orange), residual  $\bar{\phi}^{\text{res},1}$  (plain grey), 1<sup>st</sup> stage output  $\phi^{\text{res},1}$  (dotted-dashed blue) and 2<sup>nd</sup> stage residual signal  $\phi^{\text{res},2}$  (plain red).

*Right*: Simulated modal residual variances as a function of the KL modes for 2 stage CAO systems with standard 2<sup>nd</sup> stage integrator control with and without ‘woofer-tweeter trick’ and 2<sup>nd</sup> stage LQG.

## References

1. M. Kasper & *al*, “PCS — A Roadmap for Exoearth Imaging with the ELT,” *The Messenger* (2021).
2. N. Cerpa-Urra, M. Kasper, C. Kulcsár, H-F. Raynaud, C. Taïssir Heritier “Cascade adaptive optics: contrast performance analysis of a two-stage controller by numerical simulations,” *J. of Astronomical Telescopes, Instruments, and Systems*, 8(1), 019001 (2022).
3. R. Conan and C. Correia, “Object-oriented Matlab adaptive optics toolbox,” in *Adaptive optics systems IV*, 9148, 91486C, International Society for Optics and Photonics (2014).