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Want to improve your cascade 2-stage AO system?
Turn it into a high-performance woofer-tweeter!

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Abstract:
2-stage cascade AO (CAO) systems enable to improve wavefront correction for high-contrast exoplanet imaging. However, 1st stage residuals entering the 2nd stage exhibit see-saw high-frequency oscillations. We propose a compensation procedure which turns the CAO into a woofer-tweeter configuration, thus significantly improving overall performance.

OCIS codes: (010.1080) Active or adaptive Optics; (010.1330) Atmospheric turbulence

1. 2-stage XAO architecture for improved exoplanet imaging

High-contrast imaging (HCI) with adaptive optics (AO) provides direct images of extra-solar planets. For optical and near-IR HCI, the AO-corrected residual halo stellar flux is the main source of noise. In order to obtain great contrast sensitivity for exoplanet imaging at small angular separations, it is therefore crucial to minimize this residual halo mainly due to AO loop delay. A straightforward approach to reduce this temporal delay would be to run these AO systems faster. A candidate architecture for future ultra-fast Extreme-AO (XAO) systems for high-contrast imaging is a cascade adaptive optics (CAO) system with two stages [1]. In a 2-stage CAO system a 1st stage running at a sampling frequency F_1 corrects for the incoming turbulent phase, producing a residual phase that enters a faster 2nd stage running at a sampling frequency $F_2 = n_{\text{os}} F_1$, where n_{os} is the CAO oversampling factor.

The residual entering the 2nd stage is the sum of the oversampled version of the 1st stage residual $\bar{\phi}^{\text{res},1}$ and of the inter-sampling fluctuations of the incoming turbulence ϕ, *i.e.* its averages over successive 1st stage sampling periods $1/F_1$. This correction plus upsampling procedure generates a 2nd stage input signal which exhibits a distinctive pattern of high-frequency see-saw fluctuations, even when the original incoming disturbance ϕ is a low-frequency signal. Figure 1-Left shows these signals for a 2-stage system with $F_1 = 1$ kHz and $F_2 = 4$ kHz (so that $n_{\text{os}} = 4$), when ϕ is a sinusoidal signal at $f_0 = 40$ Hz and where the two stages use standard integral action controllers. Even in this simple case, these high-frequency see-saw fluctuations are visibly not properly compensated by the 2nd stage controller. A more detailed time- and frequency-domain analysis of this partial compensation with oversampling issue is presented in [2]. As it turns out, compensating this quasi-cyclical 1st stage residual with a linear time-invariant controller is very difficult. (Indeed, its temporal statistics are not time-invariant, so that it cannot be accurately modeled as a stationary process.)

2. The woofer-tweeter trick

A simple way to circumvent this problem is to turn the 2-stage system into an equivalent woofer-tweeter system, where an optimized control effort is computed from the higher-rate measurements provided by the 2nd stage WFS, then split between the 2 DMs. To explain how to pull off this trick, consider any linear 2nd controller implemented in standard state-space form as

$$x_{k+1} = A_u x_k + B_u y_{2,k}\,, \tag{1}$$

$$u_{2,k} = C_u x_{k+1}\,, \tag{2}$$

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where $u_{2,k}$ and $y_{2,k}$ are the 2nd stage control and WFS measurement. Assuming for the sake of simplicity that the 2nd stage operates in a linear regime and that the computational delays for both AO loops is one frame, y_2 is a function of the residual wavefront $\phi^{\text{res},2}$ which depends on corrections performed by the 2 stages, namely

$$\phi_k^{\text{res},2} = \phi_k + N_2 u_{2,k-1} + N_1 u_{1,k-n_{\text{os}}} \tag{3}$$

where $u_{1,k}$ is the oversampled 1st stage control action and N_1 and N_2 are the two DMs influence functions. In a configuration where the 2nd stage would operate alone (*i.e.*, with $u_1 = 0$), equations (1-2) would remain identical, while the residual wavefront would become $\phi_k^{\text{res},2} = \phi_k + N_2 u_{2,k-1}$. Thus, to make the cascade 2nd stage control exactly equivalent to this stand-alone version, it suffices to make these two different expressions of $\phi_k^{\text{res},2}$ equal, and thus to replace (2) by $u_{2,k} = C_u x_{k+1} - N_2^{\dagger} N_1 u_{1,k+1-n_{\text{os}}}$. This way, the control effort $C_u x$ computed by the 2nd stage controller is thus split between the 1st and 2nd stage DMs.

This woofer-tweeter trick was applied to the 2-stage configuration in [2], using the simulation package OOMAO in MATLAB [3]. The two controllers use a truncated basis of Karhunen-Loève (KL) modes. The two stages run at $F_1 = 1$ kHz and $F_2 = 4$ kHz. Figure 1-Right compares the resulting residual modal phase variances for the 1st stage integrator operating alone (green curve), a conventional 2nd stage integrator (blue curve), the same 2nd stage integrator implementing the woofer-tweeter trick (red dotted line) and a 2nd stage LQG controller based on a modal turbulence model and also using the woofer-tweeter trick (red curve). The corresponding Strehl ratios (in I band) estimated from the residual variances are respectively 51.7 %, 62.9 %, 66.1 % and 66.5 %. Thus, implementing the woofer-tweeter trick with a standard integrator increases the SR by more than 3 points. An LQG predictive control improves performance further, especially for low-order modes – which should result in a significant improvement in terms of HCI performance.

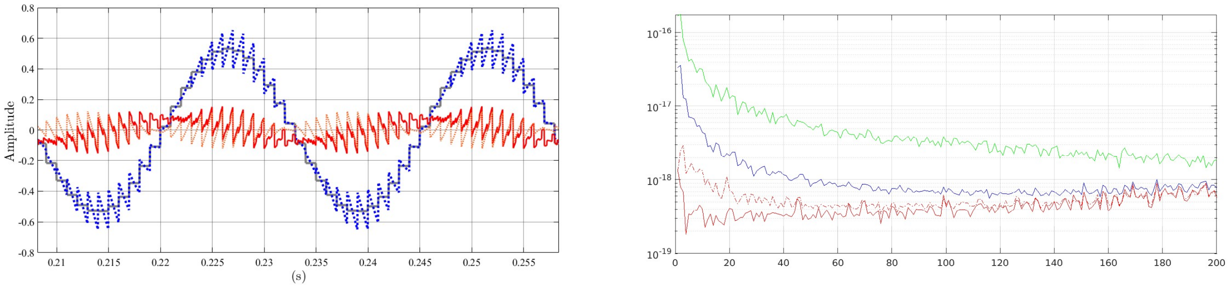

Fig. 1. *Left*: For an input sinusoidal signal of amplitude 1 at $f_0 = 40$ Hz: inter-sampling signal ϕ^{inter} (dotted orange), residual $\bar{\phi}^{\text{res},1}$ (plain grey), 1st stage output $\phi^{\text{res},1}$ (dotted-dashed blue) and 2nd stage residual signal $\phi^{\text{res},2}$ (plain red).
Right: Simulated modal residual variances as a function of the KL modes for 2 stage CAO systems with standard 2nd stage integrator control with and without 'woofer-tweeter trick' and 2nd stage LQG.

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