Interlaced phase screen and convolution operators: an efficient modeling and computational tool for ELT-sized Adaptive Optics systems

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Abstract: An interlacing procedure enabling to construct spatially invariant models of ELT-sized AO systems and atmospheric turbulence involving different spatial resolutions is presented. This approach enables to perform highly parallelizable off-line and on-line computations.

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1. Spatially invariant models and the curse of dimensionality

The control algorithms of the Adaptive Optics systems intended to equip the upcoming Extremely Large Telescopes (ELTs) will have to cope with numbers of Deformable mirror (DM) commands, wavefront sensor (WFS) measurements larger by an order of magnitude than for AO systems currently operational on existing VLT-class instruments. As a result, high-performance control algorithms based on matrix-vector multiplications simply cannot be scaled up to ELT-sized AO systems, at least using available computing technology. To overcome this 'curse of dimensionality', it has been proposed to switch to an approach based on assumptions of spatial invariance. This would enable to replace matrix-vector multiplications with Fourier-domain computations, spatial convolutions or some combination of the two techniques [1,2].

However, these spatially invariant approaches exhibit several significant drawbacks. These include the need to mitigate edge effects, but also the fact that in order to achieve efficient disturbance compensation, the incoming turbulent wavefront need to be discretized at a resolution higher than the DM/WFS spatial density. The snatch is that selecting a higher resolution makes it impossible to construct a model of the AO loop – including the DM and WFS subsystems – which would be both accurate and spatially invariant. In addition, in tomographic configurations, high performance AO control may require different spatial resolutions at different altitudes [3].

In this contribution, we show how to overcome this conundrum by resorting to an interlacing procedure which enables to combine stochastic disturbance models with DM and WFS models at different resolutions into a single spatially invariant model, albeit involving vectors of infinite 2D screens and matrices of spatial convolutions. These interlaced models, which are inherently very easy to parallelize efficiently, can the be used to perform a number of useful on-line and off-line calculations, such as computing optimal projections and interpolations or evaluating the DM fitting error variance.

2. Interlaced phase screens, matrix-valued convolution operators and Gaussian random fields

Consider first an infinite 2D phase screen ϕ discretized over a spatial grid with resolution $d_{\rm s}$, with $\phi(i,j)$ denoting its value at spatial coordinates $(d_{\rm s}i, d_{\rm s}j)$. Let us assume for the sake of simplicity that the DM and WFS share the same spatial resolution $d_{\rm sub} = n_{\rm s}d_{\rm s}$, so that each WFS subaperture contains $n_{\rm s}^2$ points. We first assign a tag $1 \leq k \leq n_{\rm s}^2$ to each of the $n_{\rm s}^2$ relative positions in a given subaperture. The interlaced version of ϕ, denoted as $\phi_{\rm e}$, will then be defined as a vector of n_s^2 infinite screens at DM/WFS resolution. For example, for $n_{\rm s} = 2$:

$$\phi_{\rm e}(i,j) = \begin{pmatrix} \phi_{{\rm e},1}(i,j) \\ \phi_{{\rm e},2}(i,j) \\ \phi_{{\rm e},3}(i,j) \\ \phi_{{\rm e},4}(i,j) \end{pmatrix} = \begin{pmatrix} \phi(2i,2j) \\ \phi(2i,2j+1) \\ \phi(2i+1,2j) \\ \phi(2i+1,2j+1) \end{pmatrix} \tag{1}$$

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Consider first a spatially invariant operator H at resolution d_s, so that the product $z = H\phi$ can be equivalently expressed as the convolution product $z = h * \phi$ or as the pointwise multiplication $z(\nu) = H(\nu)\phi(\nu)$in the Fourier domain. The interlaced version of h is then a square matrix of convolution operators with resolution d_pup, acting on ϕ_e. For example, when $d_\mathrm{sub} = 2d_\mathrm{s}$, the interlaced versions of the high-resolution one-step horizontal and vertical shifts S_x and S_y are respectively

$$s_{x,\mathrm{e}} = \begin{pmatrix} 0 & s_x & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & s_x \\ 0 & 0 & I & 0 \end{pmatrix} \qquad s_{y,\mathrm{e}} = \begin{pmatrix} 0 & 0 & s_y & 0 \\ 0 & 0 & 0 & s_y \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{pmatrix} \tag{2}$$

This procedure enables to handle operators which juggle between the two spatial resolutions d_s and d_pup. Thus, the under-sampling operator which extracts the phase values from the upper left corners of every subaperture will be represented by the $1 \times n_\mathrm{s}^2$ line vector $U_\mathrm{e} = \begin{pmatrix} I & 0 & \dots & 0 \end{pmatrix}$. Conversely, the DM influence function which maps a grid of commands at resolution d_sub onto a phase at the resolution of the DM command grid will be represented by an $n_\mathrm{s}^2 \times 1$ column vector $N = \begin{pmatrix} N_1 & N_2 & \dots & N_{n_\mathrm{s}^2} \end{pmatrix}^\mathrm{T}$. Finally, when ϕ is Gaussian zero-mean stationary random field, *i.e.* a 2D stationary Gaussian process with spatial covariance $\Sigma_\phi(i,j) = \mathrm{var}(\phi(p+i,q+j),\phi(p,q))$, the spatial covariance $\Sigma_{\phi,\mathrm{e}}$ of its interlaced version ϕ_e takes the form of a $n_s^2 \times n_s^2$ matrix of convolution operators at the coarser scale d_sub.

3. Optimal projection onto DM and phase interpolation, fitting error and interpolation variance evaluation

The optimal (minimum variance) projection of a high-resolution phase ϕ onto the DM is obtained as the matrix convolutive version of the standard matrix-vector orthogonal projector, namely $P_u = (N^\mathrm{t}N)^{-1}N^\mathrm{t}$, where N^t is the transpose of N. In practice, each of the n_s^2 convolution kernels in this interlaced operator can be accurately computed by evaluating their discrete-time Fourier transform over a sufficiently large square grid. Likewise, the optimal interpolator Int_opt which enables to compute the conditional expectation $Int_\mathrm{opt}\phi_{\mathrm{e},1} = \mathrm{E}(\phi_\mathrm{e}|\phi_{\mathrm{e},1})$ is given by the standard formula $Int_\mathrm{opt} = U_\mathrm{e}^\mathrm{t}\Sigma_{\phi,\mathrm{e}}(U_\mathrm{e}^\mathrm{t}\Sigma_{\phi,\mathrm{e}}U_\mathrm{e})^{-1}$.

4. Illustrative ELT-sized simulation

As an illustrative application, we compute the convolutive operators N and P_u in a scenario representative of an E-ELT AO system, with a subaperture size $d_\mathrm{pup} = 0.5$ m, a Gaussian DM influence function with coupling factor 0.3 and an oversampling factor $n_\mathrm{s} = 8$. Convolutive calculations were performed with a spatial covariance Σ_ϕ limited to an 80 m wide square domain, assuming standard Kolmogorov statistics, outer scale $L_0 = 25$ m and Fried parameter $r_0 = 12$ cm. Using the fact that the DM fitting error σ^2_fit corresponds exactly to the mean of the variances of the coordinates of the interlaced phase screen $(I - NP_uU_\mathrm{e})\phi_\mathrm{e}$, this error budget term was evaluated by generating a 80 m wide phase screen ϕ at the fine spatial resolution, interlacing it, then computing and desinterlacing $(I - NP_uU_\mathrm{e})\phi_\mathrm{e}$. The result of this Monte-Carlo simulation gives a fitting error evaluation in only a few minutes on an ordinary laptop computer without any parallelization.

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