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Monitoring squeezed collective modes of a 1D Bose gas after an interaction quench using density ripples analysis

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We investigate the out-of-equilibrium dynamics following a sudden quench of the interaction strength, in a one-dimensional quasi-condensate trapped at the surface of an atom chip. Within a linearized approximation, the system is described by independent collective modes and the quench squeezes the phase space distribution of each mode, leading to a subsequent breathing of each quadrature. We show that the collective modes are resolved by the power spectrum of density ripples which appear after a short time of flight. This allows us to experimentally probe the expected breathing phenomenon. Our results are in good agreement with theoretical predictions which take the longitudinal harmonic confinement into account.

The out-of-equilibrium dynamics of isolated quantum many-body systems is a field attracting a lot of interest. On the theoretical side, many questions are currently investigated and debated. Whether and how the system relaxes towards an equilibrium state is in particular the subject of intense work and the role of integrability is still not completely clear. A particular focus has been devoted to the case of sudden quenches where the system is brought out-of-equilibrium by a sudden change of a Hamiltonian parameter \[ g, \kappa g, \] and in particular the case of an interaction quench. On the experimental side, the subsequent evolution of correlation functions has been investigated in several experiments \[ 4, 5 \] . In particular in \[ 5 \] , a light cone effect has been observed in the first order correlation function after splitting a 1D Bose gas into two copies. After the light-cone extends over a few correlation lengths the system showed thermalization \[ 6 \] . However, this apparent thermalization may conceal an underlying non-equilibrium behavior. This is especially true for integrable or quasi-integrable systems where the system might not relax towards a Gibbs ensemble. In a recent experiment, long term non-equilibrium dynamics has been revealed by a revival phenomenon \[ 7 \] , although the dynamics at play before the revival were not visible in the first order correlation function. Finding proper observables revealing the dynamics is thus a key point for investigating out-of-equilibrium phenomena.

In this paper, we investigate the out-of-equilibrium dynamics following a sudden quench of the interaction strength in a quasi-1D Bose gas with repulsive interactions. While the complete treatment is tremendously difficult and has been the subject of several theoretical studies \[ 8, 10 \] , the problem is greatly simplified if one can rely on a linearized approach. For each collective mode one then expects the quench to produce a squeezed phase space distribution, leading to a subsequent oscillation of each quadrature width — a breathing behavior. These oscillatory dynamics are \textit{a priori} not visible if one considers observables mixing different modes, such as the one-body correlation function. The dephasing between modes then leads to an apparent thermalization, occurring on a relatively short time. We show in this paper that individual collective modes may be monitored using an analysis of the density ripples appearing after short time of flight. Experimentally, we observe the breathing of squeezed modes, revealing the mechanism at play after an interaction quench. In the first part of the paper, we present the theoretical framework predicting the expected squeezing and subsequent dynamics of each collective mode. We then expose why density ripples give access to the dynamics of individual modes. Finally, we present our experimental results and compare them to the derived theoretical predictions.

The relevant physics can be understood by considering a 1D homogeneous Bose gas, of length \( L \) and density \( n_0 \), with particles of mass \( m \) interacting with repulsive contact interactions \( g \delta(z_i - z_j) \) at a temperature \( T \). At \( t = 0 \), \( g \) is suddenly changed from \( g_i \) to \( g_f = (1 + \kappa)g_i \), where \( \kappa \) is the quench strength. Within the quasi-condensate regime, density fluctuations are strongly reduced \( |\delta n(z)| \ll n_0 \) and phase fluctuations occur on large length scales, such that the...
Hamiltonian of the system can be diagonalized using the phase-density representation of the field operator \( \Psi(z) = \sqrt{n_0 + \delta n(z)} \exp(i \theta(z)) \) and the Bogoliubov procedure [11]. The obtained linearized modes correspond to Fourier modes. For each wave-vector \( q \), the dynamics is governed by the harmonic oscillator Hamiltonian [12]

\[
H_q = A_q \hat{n}_q^2 + B_q \hat{\theta}_q^2 = \hbar \omega_q \left( \frac{\hat{n}_q^2}{2} + \frac{\hat{\theta}_q^2}{2} \right)
\]

where the canonically conjugated hermitian operators \( n_q \) and \( \theta_q \) are the Fourier components [13] of \( \delta n \) and \( \theta \) and where the reduced variables are defined by \( \tilde{n}_q = n_q(B_q/(4A_q))^{1/4} \) and \( \tilde{\theta}_q = \theta_q(A_q/(4B_q))^{1/4} \). For wavevectors \( q \) much smaller than the inverse healing length \( \xi^{-1} = \sqrt{m \delta n_0}/\hbar \), the excitations are of hydrodynamic nature [14]. Their frequency is \( \omega_q = cq \), where the speed of sound is \( c = \sqrt{n_0 \delta n_0 \mu/m} \), and the Hamiltonian’s coefficients are \( B_q = \hbar^2 \tilde{c}^2 n_0/(2m) \) and \( A_q = \tilde{c}^2/(2n_0) \). Here \( \mu(n) \) is the equation of state of the gas relating the chemical potential \( \mu \) to the linear density, which reduces to \( \mu = gn \) for pure 1D quasi-condensate. For a given \( q \), the dynamics of the quenched harmonic oscillator is represented Fig. [1]. Before the quench the phase space distribution is the one of a thermal state: an isotropic Gaussian in the \((\tilde{\theta}_q, \tilde{n}_q)\)-plane. The quench affects \( A_q \) while \( \tilde{\theta}_q \) and \( n_q \) do not have time to change. The variances thus become \( \langle \tilde{\theta}_q^2 \rangle_{t=0^+} = (1 + \kappa)^{1/4} \langle \tilde{\theta}_q^2 \rangle_{t=0^-} \) and \( \langle \tilde{n}_q^2 \rangle_{t=0^+} = \langle \tilde{n}_q^2 \rangle_{t=0^-} / (1 + \kappa)^{1/4} \) [15]. The subsequent evolution is a rotation in phase space at a frequency \( \omega_q \) leading to a breathing of each quadrature. In particular

\[
\langle \tilde{\theta}_q^2 \rangle = \langle \tilde{\theta}_q^2 \rangle_{t=0^+} (1 + \kappa \sin^2(q \delta t)),
\]

where the initial value \( \langle \tilde{\theta}_q^2 \rangle_{t=0} \) is the thermal prediction \( \langle \tilde{\theta}_q^2 \rangle = mk_BT/(\hbar^2 n_0 q^2) \) [16].

Probing the non equilibrium dynamics following a quench is not straightforward, especially concerning the choice of observable. Since density fluctuations are very small within the quasi-condensate regime, it is more advantageous to probe the phase fluctuations. For this purpose, one way is to investigate the one-body correlation function \( g_1(z) = \langle \Psi(z)\Psi(0) \rangle \), which can for instance be measured via its Fourier transform, the momentum distribution [17]. For distances much larger than \( \xi \), density fluctuations give a negligible contribution and, for Gaussian distributions of \( \theta \), the Wick theorem gives \( g_1(z) \simeq n_0 e^{-(\delta c(z) - \theta_0)^2/2} \). However since phase fluctuations are large in a quasi-condensate, the exponential cannot be linearized and \( g_1(z) \) mixes contributions from all Bogoliubov modes [18], preventing the observation of the squeezed dynamics. In fact, the linearized model above predicts the light-cone effect on the \( g_1 \) function: \( g_1(z) \) changes from its initial exponential decay \( \exp(-|z|/t_c') \), where \( t_c' = 2\hbar^2 n_0/(mk_BT) \), to an exponential decay with a new correlation length \( t_c = 2t_c'/(\kappa + 2) \) for \( z < 2\delta_t \). The breating of each squeezed Bogoliubov mode is not transparent here. Moreover, for times larger than a few \( t_c'' = t_c'/c \), the \( g^{(1)} \) function essentially reaches the form expected for a thermal state at a temperature \( T_f = T(\kappa + 2)/2 \), and the ongoing dynamics is hidden. In this paper we use the density ripples analysis to reveal the non equilibrium dynamics of the gas by probing the breathing of each mode.

Density ripples appear after switching the interactions off and waiting for a free evolution time \( t_f \) (time-of-flight), during which phase fluctuations transform into density fluctuations [19]. The analysis of these density ripples has been used as thermometry [20, 21], and to investigate the cooling mechanism [22]. Consider the power spectrum of density ripples \( \langle |\rho(q)|^2 \rangle \), where \( \rho(q) = (1/\sqrt{2}) \int dz (n(z, t_f) - n_0) e^{iqz} \). Propagating the field operator during the time of flight and assuming translational invariance we obtain [23]

\[
\langle |\rho_{00}(q)|^2 \rangle = \int dx e^{-iqx} f(q, x) n_0^2 \theta_q^2 \sin^2 \left( \frac{\hbar c q t_f/2m}{2} \right),
\]

where

\[
f(q, x) = \langle \psi^+(0) \psi(-\hbar q t_f/m) \rangle \propto n_0^2 \exp[i(\theta_0 - \theta(-\hbar q t_f/m) + \theta(x - \hbar q t_f/m) - \theta(x))],
\]

averages in Eq. (1) being taken before the time of flight. The function \( f \) involves only pairs of points separated by \( \hbar q t_f/m \). For small wave vectors \( q\hbar t_f/m \ll l_c \), the phase difference between those points is small and one can expand the exponential. To lowest order, assuming uncorrelated distributions for each mode \( q \) and vanishing mean values, we then find

\[
\langle |\rho_{00}(q)|^2 \rangle = 4n_0^2 \langle \theta_q^2 \rangle \sin^2 \left( \frac{\hbar c q t_f/2m}{2} \right),
\]

showing that, for low lying \( q \), the density ripples spectrum directly resolves the phase quadrature \( \langle \theta_q^2 \rangle \) of individual Bogoliubov modes [21]. The proportionality between \( \langle |\rho_{00}(q)|^2 \rangle \) and \( \langle \theta_q^2 \rangle \) implies that \( \langle |\rho_{00}(q)|^2 \rangle \) oscillates according to Eq. [3] when varying the time \( t \) after the quench. Density ripples are thus an ideal tool to investigate the quench dynamics.

In typical experiments, atoms are confined by a smooth potential \( V(z) \), generally harmonic, which complicates the picture. However, if the confinement is weak enough and for wavelengths much smaller than the system’s size, one can use the above results for homogeneous systems within a local density approximation (LDA). More precisely, \( \hat{\rho}(q) = \int dz \delta n(z, t_f) e^{iqz} \) fulfills \( \langle |\hat{\rho}(q)|^2 \rangle = \int dz \langle |\rho_{00}(z, q)|^2 \rangle \) where \( n_0(z) \) is the density profile. The latter can itself be evaluated within the LDA using the gas equation of state and the local chemical potential \( \mu(z) = \mu_0 - V(z) \). Injecting Eq. [2] and Eq. [6] into the LDA integral above, we find

\[
\langle |\hat{\rho}(q)|^2 \rangle/\langle |\rho(q)|^2 \rangle = 1 + \kappa \mathcal{F}(cq t_f),
\]
where $c$ is the speed of sound after the quench evaluated at the trap center and $\mathcal{F}$ only depends on the shape of $V(z)$. The explicit expression of $\mathcal{F}$ in the case of a harmonic potential is given in [25]. Since the central part of the cloud gives the dominant contribution, $⟨|\tilde{\rho}(q)|^2⟩$ still presents an oscillatory behavior as a function of $t$, $\mathcal{F}(\tau)$ being close to $\sin^2(\tau)$. The spread in frequencies originating from the inhomogeneity in $n_0$ is however responsible for a damping, which is a pure dephasing effect.

The experiment uses an atom-chip set up [26] where atoms are magnetically confined using current-carrying micro-wires. The transverse confinement is provided by three parallel wires carrying AC-current modulated at 400 kHz, which renders the magnetic potential insensitive to wire imperfections and, allows for independent control of the transverse and longitudinal confinements. We perform radio frequency (RF) forced evaporative cooling until we reach the desired temperature. We then increase the frequency of the RF by 60 kHz, providing a shield for energetic three-body collision residues and wait for 150 ms relaxation time [27]. The clouds contain a few thousands atoms, in a trap with a transverse size at larger $na$ being the initial power spectrum.

The interaction strength quench therefore amounts to ramping the transverse trapping frequency $\omega_{\perp}$ from its initial value $\omega_{\perp,i}$ to its final value $\omega_{\perp,f} = (1 + \kappa)\omega_{\perp,i}$ within a time $t_f$, typically of the order of 1 ms. This time is long enough for the transverse motion of the atoms to follow adiabatically but short enough so that the quench can be considered as almost instantaneous with respect to the probed longitudinal excitations [25]. In order to avoid dynamics of the mean profile and modification of the Bogoliubov wavefunctions [25], we simultaneously adapt the longitudinal trapping frequency, such that the Thomas-Fermi radius stays constant.

In order to probe density ripples, we release the atoms from the trap and let them fall under gravity for a time $t_f = 8$ ms before taking an absorption image. The transverse expansion, occurring on a time scale of $1/\omega_{\perp}$, ensures the effective instantaneous switching off of the interactions with respect to the probed longitudinal excitations. The density ripples produced by the phase fluctuations present before the free fall are visible in each individual image, as seen in Fig. (2)(a). From the image, we record the longitudinal density profile $\rho(z,t_f)$ and its discrete Fourier transform $\tilde{\rho}(q)$, which is obtained in the same conditions with atom number fluctuations smaller than 10%. From this data set, we then extract the power spectrum $⟨|\tilde{\rho}(q)|^2⟩$. We note $ ⟨|\tilde{\rho}(q)|^2⟩$, the power spectrum obtained before the quench and a typical spectrum is shown in Fig. (2)(b). We chose to normalize the momenta by $qR_{TF}$; since the Fourier distribution of the $i$th Bogoliubov mode of a 1D quasi-condensate is peaked at $k_i \simeq i/R_{TF}$ [25], the $x$-axis roughly corresponds to the mode index. The predicted power spectrum $⟨|\tilde{\rho}(q)|^2⟩_{\text{th}}$ is computed using the LDA and analytical solution of Eq. (3) for thermal equilibrium [24][31]. This expression is peaked around $kR_{TF} \simeq \sqrt{m/(\hbar T)}R_{TF} \simeq 50$. For comparison with experimental data, we take the imaging resolution into account by multiplying $⟨|\tilde{\rho}(q)|^2⟩_{\text{th}}$ with $e^{-k^2\sigma^2/2}$ where $\sigma$ is the rms width of the impulse imaging response function, assumed to be Gaussian. The experimental data ultimately compared well with the theoretical predictions, as shown in Fig. (2)(b), where $T$ and $\sigma$ are obtained by fitting the data [32].

We then investigate the dynamics following the quench of the interaction strength by acquiring power spectra of density ripples at different evolution times $t$ after the quench. We typically acquire power spectra every 0.5 ms, over a total time of 5 ms. A few raw spectra are shown in Fig. (4)(b), for a quench strength $\kappa = 2.0$. At first sight
the power spectra seem erratic. In order to reveal the expected oscillatory behavior of each Fourier component we introduce, for each wavevector \( q \) of the discrete Fourier transform, and each measurement time \( t \), the reduced time \( \tau = c q t \), where \( c \) is evaluated for the central density, and compute \( J(q, \tau) = \langle |\hat{\rho}(q)|^2 \rangle(t)/\langle |\hat{\rho}(q)|^2 \rangle_i \). We restrict the range of \( q \) values to \( 10 < q R_{TF} < 40 \), to ensure both the condition \( q \hbar c/m \ll L_c \) and the validity of the LDA. On the resulting set of sparse data, shown in the inset of Fig. (3), an oscillatory behavior appears, despite noise on the data. To combine all the data in a single graph, we perform a “smooth” binning in \( \tau \), i.e., we compute, for any given reduced time \( \tau \), the weighted averaged of the data with a Gaussian weight function in \( \tau \) of width \( \Delta = 0.31 \) : namely we compute \( J(\tau) = \sum_{q} J(q, \tau) e^{-\left(\tau - \tau_{q}\right)^2/(2\Delta^2)}/\sum_{q} e^{-\left(\tau - \tau_{q}\right)^2/(2\Delta^2)} \), where the sum is done on the data set. The function \( J(\tau) \), shown in Fig. (3) shows a clear oscillatory behavior.

The temporal behavior of the data is in good agreement with the predicted one: both the frequency and the observed damping are in agreement with the predictions. The amplitude of the experimental oscillations on the other hand are significantly smaller than the predictions, and in Fig. (3) we plot the theoretical predictions for quench strengths twice as small as the experimental ones. Moreover, for a given quench strength, the observed amplitude depend on the initial transverse frequency, in contradiction with the theoretical model. Several effects leading to a decrease of the oscillation amplitude are discussed in the [25]. However, they account only partially to the observed amplitude reduction.

In conclusion, analyzing density ripples, we revealed the physics at play after a sudden quench of the interaction strength in a quasi-1D Bose gas, namely the breathing associated to the squeezing of each collective mode. The observed out-of-equilibrium dynamics continues for times larger than \( t_{th}^0 \), for which the \( g_1 \) function essentially reached its asymptotic thermal behavior [33]. This can be seen in the inset of Fig. (3) where data corresponding to \( t > t_{th}^0 \), shown in red, still present an oscillatory behavior. This clearly underlines the power of density ripple analysis to unveil out-of-equilibrium physics. The observed damping is compatible with the sole dephasing effect due to the longitudinal harmonic confinement. At later times, the discreteness of the spectrum and its almost constant level spacing is expected to produce a revival phenomenon. Its observation might however be hindered by a damping of each collective mode due to non-linear couplings. Such a damping occurs, despite the integrability of the 1D Bose gas with contact repulsive interactions, because the Bogoliubov collective modes do not correspond to the conserved quantities. A long-lived non-thermal nature of the state produced by the interaction strength might be revealed either by observing excitations in both the phononic regime and the particle regime of the Bogoliubov spectrum [34], or, ideally, in finding a way to access the distribution of the Bethe-Ansatz rapidities.

We repeat the experiment for different quench strengths \( \kappa = (\omega_{\perp}/\omega_{\perp,i} - 1) \in \{0.3, 3.5\} \), and initial trapping oscillation frequencies \( \omega_{\perp} = \{3, 1.5\} \) kHz. The oscillatory behavior is present in all cases as shown in Fig. (2). We compared the observed oscillations with the theoretical predictions from the linearized model, Eq. (5).
Isolating the contribution of individual modes to the phase space area is preserved, one quadrature being squeezed, while the other is anti-squeezed.

For quasi-1D gases the hydrodynamic condition is re-established in [29].

For each positive frequency value, one has 2 Fourier components: \( \hat{n}_{q,c} = \sqrt{2/L} \int dz \sin(z) \cos(qz) \) and \( \hat{n}_{q,s} = \sqrt{2/L} \int dz \sin(z) \sin(qz) \), with similar expressions for \( \theta \). We omit the subscript \( c \) or \( s \) in the text for simplicity.

For quasi-1D gases the hydrodynamic condition is replaced by \( \omega_q \ll \omega_\perp \).

The phase space area is preserved, one quadrature being squeezed, while the other is anti-squeezed.

For the \( q \) values considered, \( \omega_q \ll k_BT \) and the Raithley-Jeans approximation holds.

Isolating the contribution of individual modes to the function \( g_1(z) \) requires looking at the Fourier transform of \( \ln(g_1(z)) \), which requires large detection dynamics.

[6] The degrees of freedom corresponding to antisymmetric variables showed thermalization, while the symmetric degrees of freedoms were at another temperature. The system was thus pre-thermalized, and true thermalization between symmetric and antisymmetric degrees of freedom occurs at longer times.
[13] For each positive \( q \) value, one has 2 Fourier components: \( \hat{n}_{q,c} = \sqrt{2/L} \int dz \sin(z) \cos(qz) \) and \( \hat{n}_{q,s} = \sqrt{2/L} \int dz \sin(z) \sin(qz) \), with similar expressions for \( \theta \). We omit the subscript \( c \) or \( s \) in the text for simplicity.
[14] For quasi-1D gases the hydrodynamic condition is replaced by \( \omega_q \ll \omega_\perp \).
[15] The phase space area is preserved, one quadrature being squeezed, while the other is anti-squeezed.
[16] For the \( q \) values considered, \( \omega_q \ll k_BT \) and the Raithley-Jeans approximation holds.
[18] Isolating the contribution of individual modes to the function \( g_1(z) \) requires looking at the Fourier transform of \( \ln(g_1(z)) \), which requires large detection dynamics.
[23] For consistency we rederive this expression, first established in [19], in [25].
[24] In Eq. (5), \( \langle \theta_i^2 \rangle = (\langle \theta_{q,c,i}^2 \rangle + \langle \theta_{q,s,i}^2 \rangle) / 2 \) where \( \theta_{q,c} \) and \( \theta_{q,s} \) are the cosine and sine Fourier components, which fulfill \( \langle \theta_{q,c,i}^2 \rangle = \langle \theta_{q,s,i}^2 \rangle \) for translationally invariant systems.
[26] The experiment is described in more detail in [17].
[27] We are aware that this initial state does not necessarily represent a thermal state [4]. However, the density ripples analysis only probes phonons, and their distribution is consistent with thermal equilibrium.
[30] The box \( L \) is chosen to be about twice the size of the cloud.
[31] Note that we did not use Eq. (5) to compute \( \langle |\hat{\rho}(q)|^2 \rangle_{\text{th}} \) since, for \( q R v > 0 \), \( h q^2 / m < 0.5 \) and the approximation of Eq. (5) overestimates the density ripples by about 30%.
[32] The transverse size of the cloud after the time-of-flight is comparable to the depth of focus of the imaging system and depends on the transverse confinement. We thus expect slightly different optical resolutions, \( \sigma_i \) and \( \sigma_f \) for data taken before and after the quench respectively. We correct for this effect to make quantitative comparison of data taken before and after the quench.
[33] At a time \( t = t^q_{\text{th}} \) the \( q \) function has reached the thermal value \( e^{-|z|/l_t^q} \) for all \( z < 2l_t^q \), the deviation from this thermal state being restricted to long distances where \( g_1(z) < e^{-2} \approx 10\% \).