Resonators Liberated from Dispersion: Broad Periodic Waveguides Operating at the Littrow Regime

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ABSTRACT

We show that broad periodic waveguides operating at the Littrow regime can form series of modes with a negative dispersion in their spacing (d*FSR* / dω > 0, *FSR* = Free Spectral Range), thanks to samples with extended range. We provide an analysis of this experimental FSR based on a nearly analytical consideration. For frequency combs, a uniform FSR (dispersion free) can be attained at any desirable frequency in the range λ = 1.5 – 4.3 μm for a silicon resonator.

Keywords: Broad waveguides, anomalous dispersion, dispersion compensation, grating coupling.

1. PRINCIPLE OF NEGATIVE DISPERSION IN LITTROW RESONATORS

We have recently introduced broad periodic waveguides operating in the Littrow regime as a special kind of resonators [1-5]. While our initial scope was to study longer guides offering the equivalent of multiple cavities with split resonances and more room for attaining nonlinear tuning, we realised that a basic properties of the "localised" modes of these guides is to have an intrinsic and strong anomalous dispersion: the free-spectral range (FSR) increases with frequency (d*FSR*/dω >0), unlike the case of a standard Fabry-Perot.

Since frequency combs have a considerable importance, and since their generation from a cw wave entails a nonlinear (FWM) cascade of energy through modes whose spacing should be as even as possible [6], a so-called "liberated from dispersion" situation, we believe that applying our resonators to frequency combs may offer a welcome extra freedom in the design of these demanding devices[7-10]: one often has no choice of optogeometric parameters in resonators to obtain dispersion compensation, for instance no choice of guide width or height in guided optics microrings.

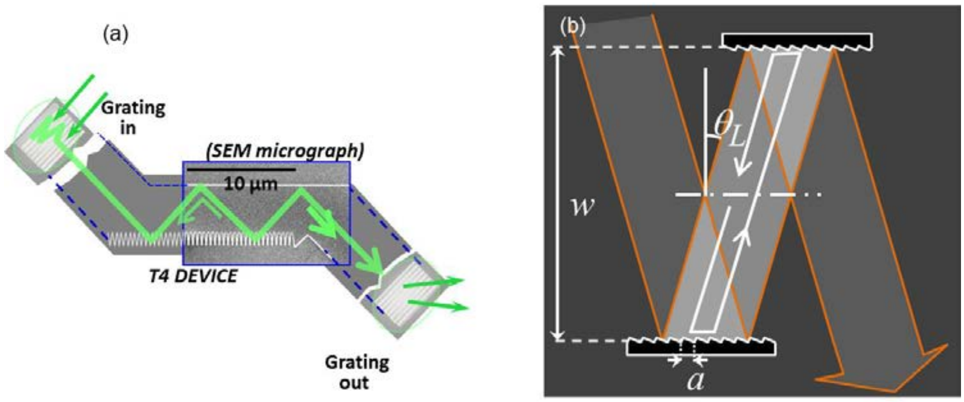

Figure 1: (a) typical "Littrow" resonator, made on SoI ; (b) unfolded scheme of a similar resonator.

First of all, it is instructive to depict our samples in true form (Fig. 1a), and to give an "unfolded" scheme of their operation (Fig. 1b), where their analogy with a modified Fabry-Perot is better seen [11].

Then, we show in Fig. 2 the construction of the bands for such a resonator: the bands of the broad periodic waveguide form themselves on the basis of the mesh of anticrossing of original waveguide bands with their folded counterparts. The basic idea is that since the spacing of these well-known branches of a non-periodic waveguide [$(\omega/c) = n^{-1}(k_x^2 + (m\pi/a)^2)$ for the *m*-th branch in index *n*] tends to increase with frequency, so do the modes of the broad periodic waveguide, whose essential behaviour can be shown to be hyperbola $\omega_H(k_x)$ centered at $k_x = \pi/a$ (Fig. 2, the mesh of crossing and the deduced hyperbola forming between).

We also show, in Fig. 2, two ways of addressing the resonances which lead to quite different slope of anomalous dispersive FSR evolution, dFSR / dω (by the same token, this makes it obvious that there is a limit to the oversimplified vision of these waveguides as localized resonators). In Fig. 2a, much as in Fig. 1a, the resonator is addressed at θ = 45°. Hence, when varying the frequency, the injected wavevector k_x varies along a light line dictated by this angle $k_x = \sin\theta(\omega/c)$. A formally simpler situation is depicted in Fig. 2b: Thanks to an extra grating of double period, an injection can easily be made from outside at normal incidence [11]. Then the tilted ray x-component of the wavevector is simply $k_x = \pi / a$, and the relevant spacing in the band-structure diagram is the one at this Brillouin zone edge, which also follows that of ordinary waveguide modes.

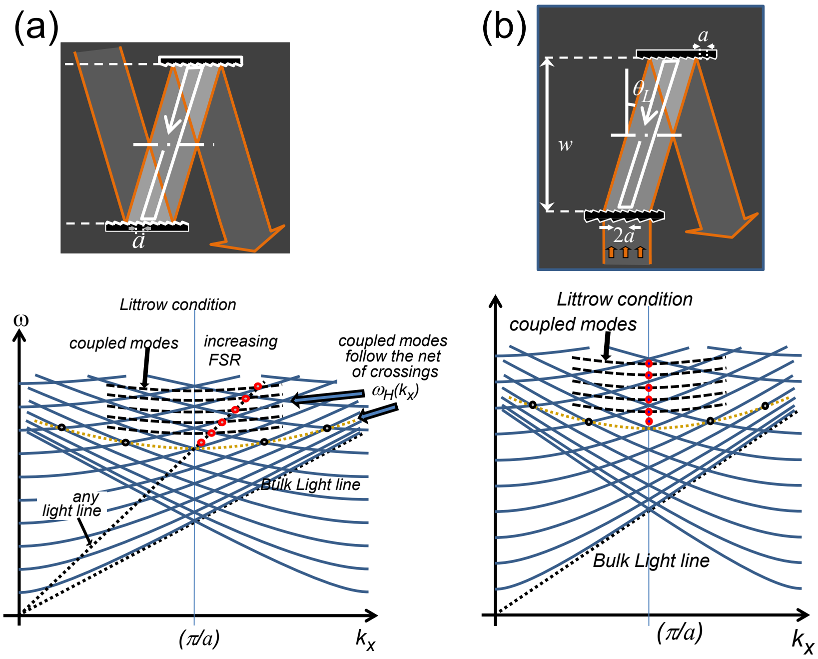

Figure 2: (a) resonator and schematic band structure for excitation at Littrow or near-Littrow off-angle, causing spread in corresponding k-values for different frequencies, and thus crossing the hyperbola of the broad waveguide dispersion $\omega_H(k_x)$ in the way indicated;(b) same for injection by the coupling grating at the bottom of the scheme. The mesh of intersections only becomes larger at higher temperature.

2. PREDICTION OF FREE SPECTRAL RANGE

Thus, some algebra outlined elsewhere [12] can suffice to get the spacing of the Littrow modes at $k_x = \pi / a$. To get the total dispersion, the chromatic/geometric dispersion of the underlying guiding structure should be incorporated, *i.e.* $n(\omega)$ of the basic nonperiodic waveguide should be known. The only remaining unknown is the exact dispersion brought by the grating, and we shall assume that it is not too strong in the present context, it should indeed be smooth on broad ranges of frequency.

Practically, one formulates the dispersion so as to express the order $m = m(\omega)$ as a function of ω including the geometric parameters, and interpolates ω_m at the integer m values based on a finely meshed set [11]. The simplest case of finding the crossing at $k_x = \pi / a$ (Fig. 1b) for frequency $\omega = 2\pi c/\lambda$ width W and period a reads for instance:

$$m = W\,[(2n(\omega)/\lambda)^2 - (1/a)^2]^{1/2} \tag{1}$$

For the more subtle case of Fig. 1a, we should first find the equation of hyperbola going through the crossings, by equating for instance $\omega(m + \Delta m,\ \pi/a - \delta k)$ and $\omega(\ m - \Delta m,\ \pi/a + \delta k,)$, with $\delta k = k - \pi / a$.

$$\omega_H^{\ 2} = (c/n(\omega))^2\ \{(\pi/a)^2 + (\pi m/W)^2 + \delta k^2(1+(W/ma)^2)\} = \omega_H(k_x) \tag{2}$$

We then introduce a ω-dependent δk to follow the light line, namely $\delta k = \pi(\omega / \omega_o - 1) / a$, with $\omega_o = 2\pi c / \lambda_o$ the working frequency, say 193 THz for $\lambda_o = 1.55$ µm. Given the presence of terms in m^2 and m^{-2} in Eq. (2), we get an equation in m^4 and m^2, whose coefficients depend on $\lambda = 2\pi c / \omega$:

$$m^4 + m^2\ \{(W^2/a^2) + (2\ W n(\omega)\ /\lambda)^2 + (1-\lambda_o/\lambda)^2(W/a)^2\} + (1-\lambda_o/\lambda)\ (W/a)^4 = 0 \tag{3}$$

From this second degree equation we can find m^2 vs. λ continuously along its branch with a positive root.

3. BROAD RANGE RESONATORS

The measurements are similar to those of [2], made with a tuneable source on SoI samples obtained from EpixFab [13]. The device length corresponding to Fig. 1a is called T4, while with a longer device called T6, there are two “cavity loops”. The data are treated as indicated in [11] in order to an accurate FSR trend with minimal assumptions. Fig.3a presents the spectrum of T4 and Fig. 3b the spectrum of T6, with split peaks as expected. Their main interest with respect to [2] is the fact that they are accessible on a broader range of above

12 THz, thus allowing more peaks to be compared and a clear FSR trend to be observed. The FSR observed are reported in Fig. 3c. The T4 data are the more clear ones. They confirm the agreement with the model of Fig. 1a (magenta curve), whereas the model of Fig. 1b, assuming $k_x = \pi / a$, provides only half the slope of the experimental data. The data therefore confirm that a strong anomalous dispersion is observed, that can be correctly described by the simple semi-analytical model developed above, in Eq. 3 (dotted line with solid dots), rather than Eq. 1 (solid line with smaller dots).

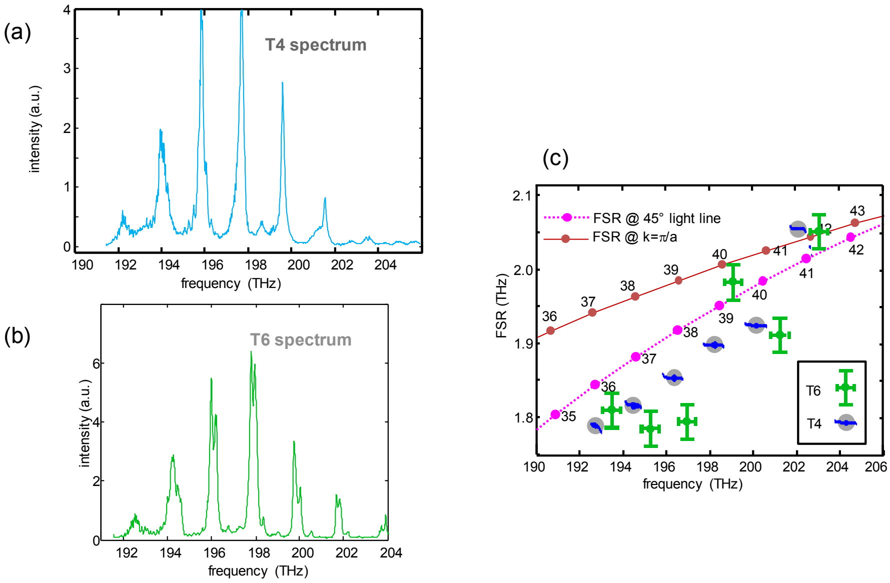

Figure 3. (a) spectrum of T4 (short) sample; (b)spectrum of T6 (long) sample (d) FSR evolution as a function of frequency for both spectra and the two theoretical lines associated to Fig.2a (magenta) and Fig.2b 'brown, top).

4. GENERAL DISPERSION-FREE DESIGN WITH GENERAL GUIDE AND GRATING

By taking into account the TE guided index dispersion $n(\omega) \equiv n_z(\omega)$ of a general slab of silicon thickness d embedded in a pair of claddings of silica (infinitely thick), we can establish [11] the dispersion and find the zero-dispersion points where the generation of frequency combs is expected to be best seeded in Littrow resonators of any period a and Littrow angle θ_L. A somewhat arbitrary order of the resonator can be chosen[11] (here around m = 500). The typical FSR dispersion is given in Fig. 4(a) for bulk silicon, for a mere waveguide of thickness d (represented in the inset of Fig. 4c), and for the Littrow geometry, following the choice of Fig. 4b, *i.e.* same as Fig. 2b because it is intrinsically a more broadband scheme, as we address slow modes at their bets points, not in the tilted parts of the hyperbola $\omega_H(k_x)$. We then identify the points of zero dispersion and the associated Littrow angles θ_L (now any angle, not 45°). The result is reported in Fig. 4c.

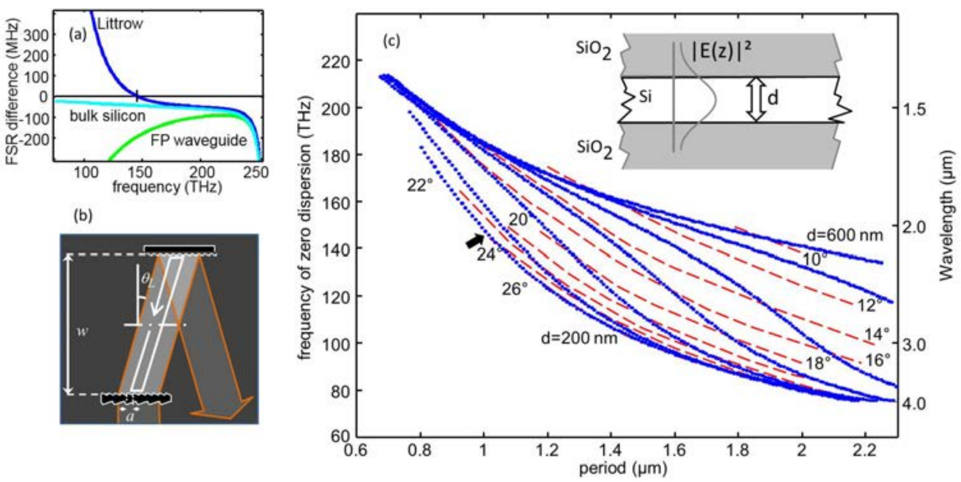

Figure 4: (a) dispersion of resonators made of bulk silicon, of a typical waveguide, and of a Littrow resonator; (b) scheme used for these Littrow resonator, operating at k_x=π/a.; (c) Locus of frequency and period for achievement of flat dispersion (dos), superimposed with contours of equi-Littrow-angles (dotted lines).

The interesting point is that thanks to tractable use of SoI type guides and easily accessible periods (0.7 µm to 2.2 µm), the range of wavelength λ = 1.5 µm to 4.0 µm can be addressed, with Littrow angles for which grating design is quite feasible (~20°), much more so than for 45° Littrow angles.

5. CONCLUSION AND PERSPECTIVES

We have shown that Littrow resonators have the potential for huge anomalous dispersion, adding an important parameter to the design tools for resonators able to produce frequency combs under cw excitation by FWM and cascade mechanism demanding locally constant FSR to be seeded. We exemplified the broad range addressable on the basis of a silicon/silica sandwich.

One issue is the actual obtainment of high finesse F for the resonator (thus high quality factor $Q = mF$). A generic answer to this question is that silicon and semiconductors are investigated quite a lot with the scope of getting high efficiency gratings [14, 15], in integrated optics as well[16], and no physical bottleneck arises with current nanofabrication methods.

Another issue is the thermal drift of such resonators. It would probably be interesting to consider *athermal* waveguide designs for this use, for instance made of silicon and polymer on top to compensate the temperature dispersion. This would make operation of frequency combs more stable. We hope to consider such aspects in the future.

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