LITTROW RESONANCES IN BROAD CORRUGATED RIBBONS : QUASI-OPTICS IN THE NO MAN'S LAND BETWEEN RESONATORS AND WAVEGUIDES

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Abstract – **We shall present the properties of « Littrow modes » in broad periodic waveguides that we recently demonstrated in SoI samples. In the band picture they correspond to multimode slow light. In the resonator picture, they mainly behave as Fabry-Perot resonators. However, there are several subtleties in their quasi-optics behaviour that we will discuss, from forbidden coupling to nonlinear conversion effects. We will stress also that they represent a general wave phenomenon.**

I. INTRODUCTION

Broadening a waveguide generally involves operating in a strongly multimode regime, with for instance multi-mode interferometers (MMI) in mind that rest on Talbot imaging. If the waveguide in question is periodic, however, the relationship between guided modes propagation constants at a given frequency becomes highly non trivial, so that any form of quasi-imaging is unlikely. Nevertheless, in the simple case of periodic parallel boundaries defining a waveguide, diffraction gives a simple picture[1]. In particular, Littrow diffraction on such a grating simplifies the situation. As pictured in Fig.1, we may then consider that the field is mainly based on four local plane waves with wavevectors symmetric against *xx'* or *yy'* symmetry.

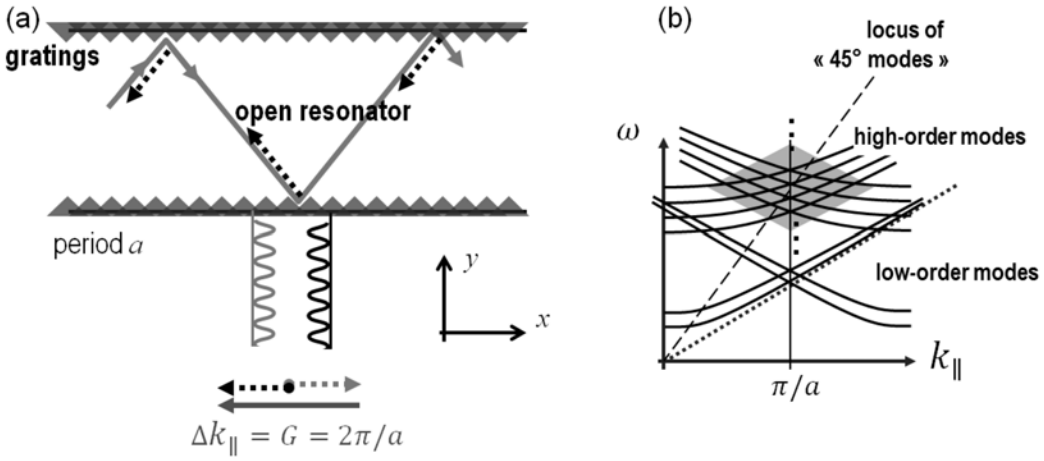

Fig. 1. (a) A broad waveguide with periodic boundaries and waves that undergo diffraction at Littrow condition ; (b) band picture, the Littrow condition corresponds to $k_{//} = \pi/a$, the Brillouin zone edge and the crossing of the two sets of modes of the underlying perfect slab waveguide.

It becomes obvious in this case that a kind of resonator can emerge as well, since a closed circuit can be made using two forward and two backwards paths. The underlying guided modes of the slab (non-corrugated) broad waveguide can also be illustrated in this case, they resemble the sum of the two waves related by the yy' symmetry, with opposite wavevectors along *x*. This rich background has been little explored in terms of quasi-optics concepts (see nevertheless Veremei's work [2]). It is indeed somewhat curious since grating pairs are of common use for dispersion purpose. On the other hand, our path to such structures has been somewhat serendipitous. We indeed used broad waveguides to make some modal interactions that we will remind briefy below,, and which are in relation with demultiplexing devices, but we were faced with the Littrow diffraction effects in waveguide when looking for the lasing condition of such systems [3]. We give a few details on the band picture and the resonator picture appropriate for such systems below, and continue with more specific issues before discussing some open issues in terms of dispersion of the obtained modes.

II. BAND PICTURE

In the band picture, it is most appropriate to start from the broad uncorrugated waveguide of width W, whose dispersion follows the well-known law $\omega=(c/n)[\ k_{//}^2+(m\pi/W)\ ]^{1/2}$, for the m-th branch. This set of hyperbola is illustrated in Fig.1b. Due to periodicity, these modes are folded in the 1st Brillouin Zone (BZ). A number of crossing points results. Those associated to the low-order modes concern not only other low-order modes near $k_{//} = \pi/a$ but also higher order modes when approaching $k_{//} << \pi/a$. These particular modal interactions termed by us "mini-stopband" have been used by us in a number of demultiplexing devices in the last decade, on InP or SoI [4-8].

However, we are more interested in the BZ edge, and at higher order modes. The rays or modes that fit Fig. 1(a) correspond to the shaded lozenge of Fig.1b. At this point, the unperturbed modes form a regular net of criss-crossing nearly equidistant modes. The crossing points exactly on the BZ edge correspond to the Littrow condition. By remarking that $\omega=(c/n)\ k=(c/n)k_{//}\ (\sin\theta)^{-1}$, lines through the origin of slope $(\sin\theta)^{-1}$ indicate the angle of operation. For instance for a 45° angle in Fig.1a, we have to draw in Fig. 1b a line with a slope $\sqrt{2}$ higher than the asymptotic (c/n) slope of the hyperbola.

The next point is that due to perturbation these modes interact and open gaps at their intersection. The generic interaction of two such parallel equidistant manifolds was studies in the context of adiabatic mode manipulation in atom physics by Demkov and Ostrowsky [9-11]. They gave an analytical formula reminded in Fig.2, showing that the modes can become perfectly flat when the interaction V is tuned to a critical value equal to $1/\pi$ times the frequency (energy) spacing, i.e. the local free spectral range (FSR). The general evolution is as illustrated below in Fig.2 :

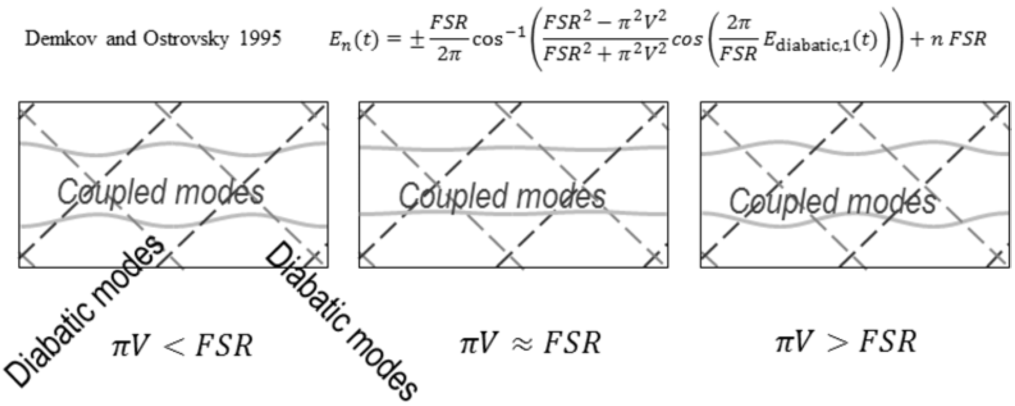

Fig. 2. Formula and illustration of the coupling of two manifolds in the Landau-Zener tunneling context by Demkow and Ostrovsky, for critical coupling (center panel) or neighbor cases (right and left). The abscissa in Demkov and Ostrovsky is time, whereas ours is wavevector, their ordinate is energy, similar to our frequency. The two manifolds correspond to diabatic (uncoupled) states. The formula above is adapted from Demkov's paper one.

The flat modes mean that light is maximally slowed down. This is of course a very interesting opportunity. In the simpler picture, attaining this critical coupling $\pi V=FSR$ amounts to get a near unity diffraction efficiency of the Littrow condition, which naturally explains that light is "stopped". There are some open questions, though, on the meaning of the situation with higher coupling $\pi V>FSR$, which is observed in actual diagrams, while it suggests higher than unity diffraction efficiency. We conjecture that a singularity has to enter the description to allow this impression: The diffraction efficiency goes through unity at some point of a parametric scan, but then an extra phase enters that gives the appearance $\pi V > FSR$, whereas the diffraction efficiency goes down.

III. Resonator picture

Here, we leave the infinitely extended (along *xx'*) modal picture and try to adopt a quasi-optics picture. We target the 45° case and we see that it is possible to build not only one elementary resonance as suggested above, but an arbitray number : using a single-sided corrugation (which avoids degeneracy issues), and based on the geometrical construction using T triangles (Fig.3), we see that we have N coupled resonances when we have a device that can be tiled with 2N+2 triangles , thus a device termed T(2N+2) (1 resonance for T4, 2 for T6, 3 for T8 etc.).

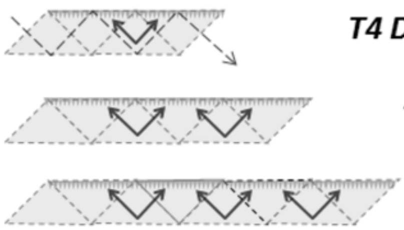

Fig. 3 Devices made of (2N+2) triangular "tiles" and supporting N resonances (N=1,2,3 for T4,T6,T8 respectively).

For such systems, we expect a behaviour similar to that of N coupled Fabry-Perot cavities : the systems are separated by identical "mirrors" (gratings with low specular order efficiency play a role akin to a good mirror) either between them or with respect to the outside. In such a case of coupled systems, we expect the normal modes and eigenfrequencies to be given essentially by diagonalizing simple matrices such as [ω_{FP} 1 0 ; 1 ω_{FP} 1 ; 0 1 ω_{FP}] (possibly with complex frequencies). A last point is to define the order *p* of the basic T4 resonator for instance. It corresponds to an optical path $\ell = 4\sqrt{2}W$. Reminding that we counted with integer *m* the mode number of Fig.1a, it is easily shown that $p=4m$. Each initial branch *m* gives two resonances as it splits into two bands (Fig.2). The other factor of two is more of geometric nature.

IV. Quasi-Optics Behaviour : forbidden coupling and nonlinear conversion effects

A. Forbidden coupling

As recently reported[12], we did measurements of such systems on SoI (Silicon-on-insulator) substrates. This point is important to make it clear that we have then a long feeding access waveguide before the device itself, which plays the role of a spatial filter.

We observed a curious trend when varying the Littrow grating strength (the *V* parameter above) through the length of the teeth : when approaching the resonance condition, the shorter devices, T4, had a sharper drop in transmission then the T6 devices (Fig.4, see [12] and S1 supplementary material therein for details). Our present explanation (Fig.5) is that the spatial filtering imposed by the long access waveguides selects the exact 45° direction. Near critical coupling, the diffraction efficiency goes to unity, and therefore at this point[13], if the angle of incidence is changed even by a small quantity, the sign of the amplitude of the "scattered wave" (here meaning the weak specular 0 order needed to get out of the resonator) is changed.

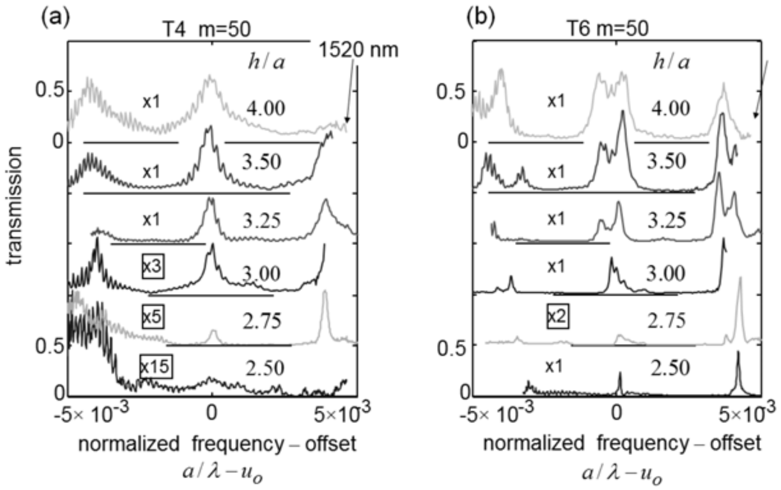

Fig. 4 Experiment of Silicon-on-insulator (SoI) open resonators (a) experimental data for T4 for various teeth height parameter, the weak signal has to be multiplied to appear on same scale ; (b) Same results for T6, note the near absence of any multiplying factor in this case.

According to the quasi-optics intuition that the mode has to map a kind of Gaussian shape to accommodate the finite resonator size (seen better in an unfolded version of the open resonator in Fig5(a)), the critical situation corresponds also to a sign change of the scattered wave along the profile of the exit waveguide. For a longer device, there is more room thanks to the larger phase space of the multiple resonators to escape this particular "forbidden coupling" situation.

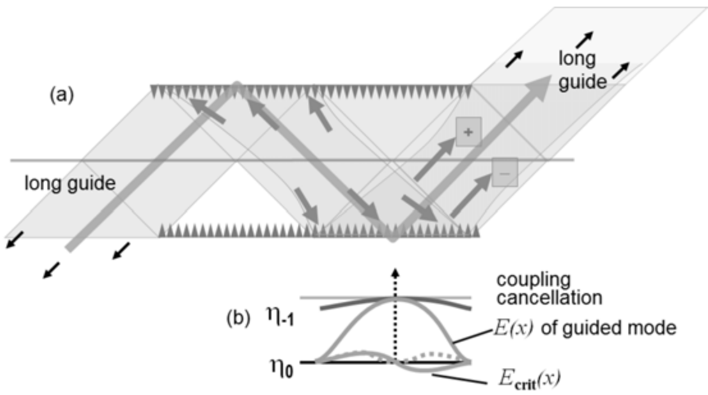

Fig. 5. (a) Intuitive picture of local rays for fitting the wave into a T4 device, with Gaussian beam spirit, and with an unfolding around the grey horizontal line (b) the profile at cirtical coupling goes through maximum (unity) diffraction efficiency into order -1, hence the 0-th order diffraction efficiency goes through a zero, and light is forbidden to couple in the fundamental mode of the long exit/input waveguide.

B. Nonlinear conversion effects

Nonlinear conversion can be seen as the local generation of a wave, with some self-diffraction then dictating its further fate. We can think of nonlinear generation in the multiple cavity (T6 or T8 case) as the scattering of a nonlinear source with the resonator environment. Here, we give some indication for the nonlinear generation by the $\chi(3)$ process of an idler wave, from a pump wave and a signal wave, a wavelength conversion process with pump in one order p, signal in order p+1 and idler in p-1 .

Our first point is that in each cavity, phase matching is nearly ensured by energy matching, since both are proportional. The mismatch effect that comes with propagation usually comes here with wave scattering into the adjacent resonators. At this stage a more detailed analysis of the eigenmodes interacting is needed. It is also important to look for a triple resonant condition, ideally, whereby the idler also falls on a resonant mode of the coupled system (for instance the cluster of two modes for Fig.4b). This analysis is under way and several positive cases exist.

V. Specific Dispersion

A last issue that we shall discuss concern the dispersive properties of such open resonators. In principle, if we focus on modes at the BZ edge $k_{//} = \pi/a$, we can guess that the first hyperbola being more crowded than the upper ones, the mode spacing should, non-conventionally, increase with increasing frequency, i.e. display anomalous dispersion, as is currently realized in the bigger cavity or devices used to compress light pulses, thans to multiple Gires-Tournois interferometers for instance.

However, when considering the FDTD simulations given in our recent work [12], we could not find a dominance of this single trend. We are currently investigating what is the role of the grating in terms of the dispersion of successive modes. In any case, we currently suspect that both “anomalous” and normal dispersion can take place in our system. Getting these dispersion aspects mastered could of course be of high interest in the case of nonlinear optics. We had suggested earlier the possibility to generate comb of frequencies matched to the successive resonances of the open resonator devices examined here. We can add that mastering dispersion of these resonances would surely help managing the subtle interplay of phenomena involved in the comb generation.

VI. Conclusion

Broad periodic waveguides operated near the Littrow condition offer a perfectly intermediate case between cavities and waveguides. We have emphasized both aspects and their respective relations. We have shown the appearance of multiple cavities of large order in an otherwise “uniform” waveguide (periodic at the wavelength scale for Littrow condition). Nonlinear optics and dispersion issues of wave scattering in a quasi-optics picture may also be a topic of large interest in the future, exploiting the rich behaviour found in these open resonators.

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