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# Rabi interferometry and sensitive measurement of the Casimir-Polder force with ultracold gases

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We show that Rabi oscillations of a degenerate fermionic or bosonic gas trapped in a double-well potential can be exploited for the interferometric measurement of external forces at micrometer length scales. The Rabi interferometer is less sensitive but easier to implement than the Mach-Zehnder, since it does not require dynamical beam-splitting or recombination processes. As an application we propose a measurement of the Casimir-Polder force acting between the atoms and a dielectric surface. We find that even if the interferometer is fed with a coherent state of relatively small number of atoms, and in the presence of realistic experimental noise, the force might be measured with a sensitivity sufficient to discriminate between thermal and zero-temperature regimes of the Casimir-Polder potential. Higher sensitivities can be reached with bosonic spin squeezed states.

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## I. INTRODUCTION

Interferometers with trapped ultracold atoms are valuable tools for the precise measurement of external forces [1]. An important achievement would be the realization of the double-well Mach-Zehnder interferometer (MZI) [2]. Its creation requires two 50:50 beam splitters implemented by a dynamical manipulation of the interwell barrier [2–5]. The phase shift is accumulated during the interaction of atoms with an external potential in the absence of interwell coupling. On the contrary, it is interesting and experimentally relevant to search for alternative interferometric schemes which could be easier to realize and therefore potentially more stable than the MZI. In this article we propose a different protocol: a double-well Rabi interferometer (RI). This can be implemented using either degenerate spin-polarized fermions or noninteracting Bose-Einstein condensates (BECs). The RI is less sensitive than the MZI but does not require any splitting or recombination processes and is potentially suitable for the estimation of forces decaying rapidly with distance. Moreover, in analogy to the MZI, but differently from previous proposals for the measurement of weak forces, the RI can reach a sub-shot-noise phase sensitivity using spin squeezed states recently created with a BEC [6]. In the scheme presented here, atoms tunnel between the two wells while acquiring a phase shift. The relative number of particles among the two wells undergoes Rabi oscillations analogous to those experienced by a collection of two-level atoms in a quasiresonant field [7]. The measurement of population imbalance as a function of time permits inference of the value of the external force as it affects both the amplitude and the frequency of Rabi oscillations. In particular, once fed with a fermionic or bosonic spin coherent state, the interferometer allows for accurate measurement of the Casimir-Polder force between the atomic sample and a surface [8]. Measurements of this force have already been performed with atoms [9] and, also, BECs [10–12]. The thermal regime of the Casimir-Polder potential has been measured at temperatures ranging from 300 to 600 K in [10]. In this article we show that the RI might allow us to distinguish between the thermal and the zero-temperature regimes of the Casimir-Polder force also in the presence of typical experimental noise.

## II. THE RABI INTERFEROMETER

We consider a degenerate gas of $N$ noninteracting atoms confined in a double-well potential along $x_1$ and in a harmonic transverse trap along $\vec{x}_\perp = (x_2,x_3)$. Taking into account only the lowest and the first excited state of the double-well potential, we introduce the operators $\hat{a}_{\mathrm{r/l},\vec{n}_\perp}$ together with the corresponding wave functions $\psi_{\mathrm{r/l}}(x_1)$. These operators annihilate a particle in the right or left well occupying a harmonic trap state labeled by indices $\vec{n}_\perp = (n_2,n_3)$. Under proper choice of commutation relations, the Hamiltonian, either for ultracold bosons (which populate only $\vec{n}_\perp = 0$) or for ultracold fermions (distributed over $N$ lowest states of the trap [13]), reads

$$\hat{H} = -E_J\hat{J}_x + \delta\hat{J}_z. \tag{1}$$

Here, $E_J$ is the tunneling energy and $\delta$ is the relative energy shift due to interaction with a position-dependent external potential $V(x_1)$ [14]. The operators $\hat{J}_x$, $\hat{J}_y$, and $\hat{J}_z$ form a closed algebra of angular momentum [15].

The goal of the Rabi interferometer is to estimate $\delta$ with the highest possible sensitivity. We consider the measurement of the population imbalance between the two modes, which corresponds to eigenvalues of the operator $\hat{J}_z$. Using the evolution operator generated by (1),

$$\hat{U}(t) = e^{-i\alpha\hat{J}_y}e^{i\omega t\hat{J}_x}e^{i\alpha\hat{J}_y}, \tag{2}$$

we obtain

$$\hat{J}_z(t,\delta) = \sin\alpha\cos\alpha(\cos\omega t - 1)\hat{J}_x - (\cos\alpha\sin\omega t)\hat{J}_y + (\cos^2\alpha\cos\omega t + \sin^2\alpha)\hat{J}_z, \tag{3}$$

where $\cos\alpha = E_J/\hbar\omega$, $\sin\alpha = \delta/\hbar\omega$, and $\omega = \sqrt{E_J^2+\delta^2}/\hbar$ is the detuned Rabi frequency.

The estimation protocol consists of measuring the population imbalance at $k$ times, $\{n\} = \{n(t_1),\ldots,n(t_k)\}$, where

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each $n(t_i)$ is averaged over $m$ independent repetitions, and $n(t_i) = \frac{1}{m}\sum_{j=1}^{m} n_j(t_i)$, where $n_j(t_i)$ is the result of a single measurement. The value of $\delta$ is estimated from a least squares fit of the theoretical curve $\langle \hat{J}_z(t,\delta)\rangle$, where the average value is calculated with the input state of the interferometer. If $m \gg 1$, the error of the estimated $\delta$ can be determined using the central limit theorem. The conditional probability for measuring a value $n(t_i)$ tends to

$$p(n(t_i)|\delta) = \frac{1}{\sqrt{2\pi}\,\Delta \hat{J}_z(t_i,\delta)/\sqrt{m}} e^{-\frac{[n(t_i)-\langle \hat{J}_z(t_i,\delta)\rangle]^2}{2\Delta^2 \hat{J}_z(t_i,\delta)/m}},$$

where $\Delta^2 \hat{J}_z(t_i,\delta) = \langle \hat{J}_z(t_i,\delta)^2\rangle - \langle \hat{J}_z(t_i,\delta)\rangle^2$. Since measurements at different times are independent, the joint conditional probability of detecting $\{n\}$ reads $p(\{n\}|\delta) = \prod_{i=1}^{k} p[n(t_i)|\delta]$. To derive the sensitivity of the fit, we notice that the least squares formula

$$\frac{\partial}{\partial \delta}\left(\sum_{i=1}^{k} \frac{[n(t_i) - \langle \hat{J}_z(t_i,\delta)\rangle]^2}{2\Delta^2 \hat{J}_z(t_i,\delta)}\right) = 0$$

coincides with maximization of the probability $p(\{n\}|\delta)$ with respect to $\delta$, that is, $\frac{\partial}{\partial\delta} p(\{n\}|\delta) = 0$. The value of $\delta$ obtained in this way is called the maximum likelihood estimator (MLE), and the corresponding error is given by the Cramer-Rao lower bound [16]:

$$\Delta^2\delta = \left(\int d\{n\} \frac{\left[\frac{\partial}{\partial\delta} p(\{n\}|\delta)\right]^2}{p(\{n\}|\delta)}\right)^{-1} = \frac{1}{\sum_{i=1}^{k} \frac{1}{\Delta^2\delta(t_i)}}, \quad (4)$$

where

$$\Delta^2\delta(t_i) = \frac{\Delta^2 \hat{J}_z(t_i,\delta)}{m\left[\frac{\partial}{\partial\delta}\langle \hat{J}_z(t_i,\delta)\rangle\right]^2}. \quad (5)$$

Since the conditions for the fit and for the MLE coincide, we conclude that the sensitivity of the former is given by Eqs. (4) and (5).

We now calculate the sensitivity using a coherent spin state (CSS) [17] as input of the RI. This state corresponds to a Poissonian distribution of particles among the two wells. For fermions, it is given by $|\mathrm{CSS}\rangle_F = \prod_{\vec{n}_\perp} \frac{1}{\sqrt{2}}(\hat{a}^\dagger_{r,\vec{n}_\perp} + \hat{a}^\dagger_{l,\vec{n}_\perp})|0\rangle$, where $|0\rangle$ is the vacuum and the product runs over the first $N$ excited states along the $(x_2,x_3)$ directions, while for bosons $|\mathrm{CSS}\rangle_B = \frac{1}{\sqrt{N!}}[\frac{1}{\sqrt{2}}(\hat{a}^\dagger_{r,0} + \hat{a}^\dagger_{l,0})]^N|0\rangle$. The CSS is an eigenstate of $\hat{J}_x$, with the eigenvalue equal to $\frac{1}{2}N$, while $\langle \hat{J}_{y,z}\rangle = 0$ and $\langle \hat{J}^2_{y,z}\rangle = \frac{1}{4}N$. The exact expression for the sensitivity at time $t_i$ is calculated with the help of Eqs. (3) and (5):

$$\Delta\delta(t_i) = E_J \frac{\sqrt{\left[\cos(\omega t_i) + \frac{\delta^2}{E_J^2}\right]^2 + \left(1 + \frac{\delta^2}{E_J^2}\right)\sin^2(\omega t_i)}}{\sqrt{mN}\left|\frac{E_J^2-\delta^2}{\hbar^2\omega^2}[\cos(\omega t_i) - 1] - \frac{\delta^2 t_i}{\hbar^2\omega}\sin(\omega t_i)\right|}.$$

It can be simplified under the assumptions $\frac{\delta^2}{E_J^2} \ll 1$ and $t_i \ll t_0 \equiv \hbar^2\omega/\delta^2$ (in Sec. III we show that these assumptions are well satisfied for typical experimental parameters). Then $\omega \simeq E_J/\hbar$, the relative population oscillates as

$$\langle \hat{J}_z(t_i,\delta)\rangle = \frac{N}{2}\frac{\delta}{E_J}\left[\cos\left(\frac{E_J t_i}{\hbar}\right) - 1\right],$$

and the sensitivity

$$\Delta\delta(t_i) = \frac{1}{\sqrt{mN}} \frac{E_J}{\left|\cos\left(\frac{E_J t_i}{\hbar}\right) - 1\right|} \quad (6)$$

scales at the shot-noise limit, $\Delta\delta(t_i) \sim N^{-\frac{1}{2}}$. The smallest error, $\Delta\delta_{\min} = E_J/2\sqrt{mN}$, is reached when $\frac{E_J t_i}{\hbar} = \pi(2j+1)$ with $j \in \mathbb{N}$.

## III. ESTIMATION OF THE CASIMIR-POLDER FORCE

In the RI, the external force perturbs the system while the atoms tunnel through the barrier. Therefore, the interferometer is best suited for measuring forces which decay on a scale of typical interwell distances of a few micrometers. As a specific application, in the following we examine the measurement of the Casimir-Polder force with a bosonic RI. We consider a BEC of $N = 2500$ $^{87}$Rb atoms trapped in a double well, with the following parameters [3]: the minima of the potential are separated by $l = 4.8$ $\mu$m and the tunneling energy equals $E_J/\hbar = 52.3$ s$^{-1}$. A surface is positioned at a distance $d$ of a few micrometers from one of the wells (see the inset in Fig. 1 for a sketch of the experimental configuration). One possibility would be to use a dielectric surface. In [10], the temperature dependence of the Casimir-Polder force was measured over the range 300 to 600 K, using a dielectric surface at distances ranging from 7 to 11 $\mu$m from the cloud. Another option would be to use a metallic plate. However, at distances of a few micrometers from magnetically or optically trapped atoms, the near-field magnetic noise originating from the metallic surface leads to decoherence and losses in the cloud, as observed in [18–20]. The underlying mechanism is the spin-flip transitions induced by either thermal currents or technical noise, creating oscillating magnetic fields [21]. Therefore, as also suggested by the measurement of trap lifetime made

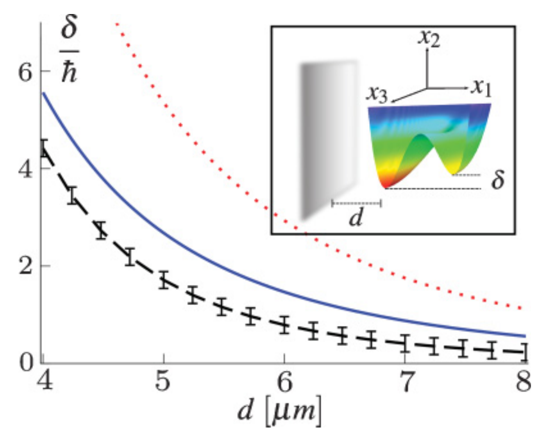


FIG. 1. (Color online) The detuning $\frac{\delta}{\hbar}$ calculated with Eq. (7) [dashed (black) line] and the corresponding sensitivity from Eq. (4) (error bars) of a fit to $k = 10$ equally spaced points in the first Rabi period with $m = 10$ measurements at each time point. The uncertainty includes the effect of residual atom-atom interactions and limited resolution of the measurement of population imbalance (see text for details). The input state is the classical spin coherent state. Also, a detuning calculated with $V^{\mathrm{th}}_{\mathrm{CP}}(x_1;d)$ for $T = 300$ K [solid (blue) line] and $T = 600$ K [dotted (red) line] is plotted. Inset: the trap configuration for measurement of the Casimir-Polder force.

with a microfabricated silicon chip [12], a dielectric might be preferable for the measurement of Casimir-Polder force proposed here, since in this case a reduction in the condensate lifetime was observed only when the surface was close enough ($<2$ $\mu$m) to reduce the trap depth. As a final comment, we note that, even when near-field magnetic noise is relevant, the coherence time at distances of 5 $\mu$m from a metallic surface can be of the order of 1 s [22], allowing for the observation a few coherent Rabi oscillations (the typical Rabi period is about 100 ms). Moreover, as discussed here, the RI can operate at a fixed optimal time within the first period, with no need for multiple oscillations.

The bosonic RI also requires the suppression of interatomic interactions (see Sec. IV for discussion of the impact of two-body interactions on the sensitivity), which can be achieved via magnetic or optical Feschbach resonances [23].

The Casimir-Polder force acts between the atoms and a surface. The exact form of the potential, given in [8], depends on the dielectric properties of the atoms and the plate, as well as on the temperature $T$ of the latter. If the thermal wavelength $\lambda_{\rm th} = \frac{\hbar c}{k_B T}$ of the photons emitted from the plate is much larger than $d$ (as it is for $d \simeq 5$ $\mu$m when $T \leqslant 100$ K [24]), the Casimir-Polder potential is well approximated by

$$V_{\rm CP}(x_1; d) = -\frac{0.24\,\hbar\, c\, \alpha_0}{\left(x_1 + \frac{1}{2} l + d\right)^4}\,\frac{\varepsilon_0 - 1}{\varepsilon_0 + 1}.$$

Here $c \simeq 3 \times 10^8$ m/s is the speed of light and $\alpha_0 = 47.3 \times 10^{-30}$ m$^3$ is the static value of $^{87}$Rb atomic polarizability. We have chosen a sapphire surface for the results given in Fig. 1, for which $\varepsilon_0 = 9.4$ is the static value of the dielectric function. The potential shifts the energy minima by

$$\delta = \int dx_1 [|\psi_r(x_1)|^2 - |\psi_l(x_1)|^2] V_{\rm CP}(x_1; d), \qquad (7)$$

and the mode functions $\psi_{r/l}(x_1)$ are given by the symmetric and antisymmetric combination of the ground and first excited state of the double-well potential. When the plate is positioned at $d = 4$ $\mu$m, then $\delta/\hbar = 4.4$ s$^{-1}$, $\frac{\delta^2}{E_J^2} = 0.007$, and $t_0 \simeq 3$ s. The period of Rabi oscillations is $\omega = 120$ ms and is much shorter than $t_0$. Therefore evaluation of the sensitivity using the approximate Eq. (6) is well justified.

If the temperature of the plate is high ($T \gtrsim 300$ K), so that the condition $\lambda_{\rm th} \gg d$ is not satisfied, the Casimir-Polder interaction is described by

$$V_{\rm CP}^{\rm th}(x_1; d) = -\frac{k_B T\, \alpha_0}{4\, \left(x_1 + \frac{l}{2} + d\right)^3}\,\frac{\varepsilon_0 - 1}{\varepsilon_0 + 1},$$

and the detuning is calculated with Eq. (7) substituting for $V_{\rm CP}(x_1; d)$ with $V_{\rm CP}^{\rm th}(x_1; d)$.

Note that, in the low-$T$ limit, the potential $V_{\rm CP}$ is proportional to $\hbar$ and $c$ and does not depend on the temperature of the surface, contrary to $V_{\rm CP}^{\rm th}$. Therefore, if the sensitivity of the RI is sufficient to distinguish between these two limits, one can discriminate between purely quantum and thermal effects. In Fig. 1, we plot the values of $\delta/\hbar$, as a function of distance $d$, calculated with $V_{\rm CP}(x_1; d)$ [dashed (black) line] and $V_{\rm CP}^{\rm th}(x_1; d)$ at $T = 300$ K [solid (blue) line] and at $T = 600$ K [dotted (red) line]. The error bars around the dashed line give the uncertainty $\Delta\delta/\hbar$ of the RI fed by a CSS by fitting to $k = 10$ points at times $t_i = \frac{2\pi\hbar}{E_J}\frac{i}{k}$ in the first Rabi period, each with $m = 10$ measurements. The sensitivity is calculated with Eqs. (4) and (6), including a realistic estimate of the experimental noise, as discussed in the next section.

## IV. SOURCES OF NOISE

Spin-polarized fermions are natural candidates for the implementation of the preceding interferometric scheme since the particle-particle interaction is naturally suppressed by the Pauli exclusion principle. Ultracold Fermi gases have been used to observe macroscopic Bloch oscillations induced by gravity in an optical lattice [26] or to perform Ramsey interferometry through Bragg diffraction [27].

In the case of bosons, the value of the $s$-wave scattering length can be strongly reduced by using Feshbach resonances. The remaining residual interaction can be taken into account by introducing an additional term $E_C \hat{J}_z^2$ into Hamiltonian (1) and calculating the first-order correction to the evolution operator (2). We checked that when $N\frac{E_C}{E_J} = 0.1$ [25], the interactions marginally spoil the sensitivity.

An important source of noise in the RI is given by the limited resolution on the population imbalance measurement. This can be taken into account by substituting the ideal probability $p(n|\delta)$ with the convolution $p_{\rm res}(n|\delta) = \sum_{n'} \mathcal{P}(n,n') p(n'|\delta)$, where $\mathcal{P}(n,n')$ gives the probability of measuring the population imbalance $n'$, given the true value $n$. We take $\mathcal{P}(n,n') = \frac{1}{\sqrt{2\pi}\sigma_{\rm res}} \exp[-\frac{(n-n')^2}{2\sigma_{\rm res}^2}]$, with a conservative value $\sigma_{\rm res} = 40$ (the population imbalance is measured with a resolution of $\pm 40$ particles). Then $\frac{\Delta\delta}{\delta}$ for the spin coherent state with 2500 atoms increases by a factor of 2 over the level of quantum noise. Yet the sensitivity is sufficient to precisely distinguish between thermal and zero-temperature regimes of the Casimir-Polder force, as shown in Fig. 1. This is one of the main results of this article.

## V. INTERFEROMETER WITH SQUEEZED INPUT STATES

So far, we have discussed the sensitivity of the RI with a coherent input state. Here we show that, keeping the number of atoms constant, a higher sensitivity can be reached with spin squeezed states, having $\xi^2 \equiv N\frac{\langle \hat{J}_z^2\rangle}{\langle \hat{J}_x\rangle^2} < 1$ [28]. The squeezing parameter $\xi^2$ is equal to 1 for a CSS and decreases when reducing the fluctuations of $\hat{J}_z$ while the coherence is kept constant, $\langle \hat{J}_x\rangle \propto N$. In Fig. 2 we show how the sensitivity (5) improves by plotting $\Delta\delta$ as a function of time for three different squeezing parameters. At $\frac{E_J}{\hbar}t = \pi$ we have $\Delta\delta = \xi\frac{E_J}{2\sqrt{N}\sqrt{m}}$ and the sensitivity scales as $N^{-\frac{1}{2}}$ for the CSS and approaches $N^{-1}$ in the limit of very strong squeezing. Figure 2 also reveals that for strong squeezing, the value of $\Delta\delta$ is small only around the optimal point. Therefore it is reasonable to focus the experimental effort around this point, instead of acquiring data distributed over the whole Rabi period. This allows us to decrease $\Delta\delta_{\rm est}$ by a factor which ranges from $\sqrt{k}$ for a strong squeezing limit to $\sqrt{8/3}$ for a coherent state. Squeezed states can be created by adiabatically splitting an interacting BEC trapped in a double-well potential, as recently experimentally

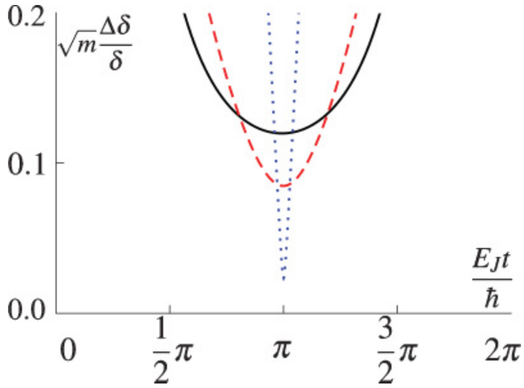


FIG. 2. (Color online) The sensitivity $\sqrt{m}\Delta\delta$ as a function of time, in units of $\delta$. Solid (black), dashed (red), and dotted (blue) lines correspond to $\xi^2 = 1.0, 0.5$, and 0.017, respectively. The sensitivities are optimal at $\frac{E_J}{\hbar}t = \pi$. Here, $N = 2500$ and $\frac{\delta^2}{E_J^2} = 0.007$.

demonstrated in [6], where a state with $\xi^2 \simeq 0.6$ for $N \simeq 2200$ particles was prepared.

## VI. COMPARISON WITH OTHER INTERFEROMETRIC SETUPS

The possibility of using cold or degenerate atoms for the measurement of forces at small distances has led to a number of proposals and experiments [10,11,29–32]. In [10], the second derivative of the Casimir-Polder potential was deduced from the shift of the frequency of the collective oscillations of a BEC in a trap put below a surface. We note that, differently from [10], the RI provides the value of $\delta$, which is related to the first spatial derivative of the perturbing potential. On the theoretical side, Refs. [29] propose to estimate the strength of the interaction between the atoms and a surface using the frequency shift of Bloch oscillations of a cold fermionic or bosonic gas in a vertical optical lattice. An important aspect of this proposal is the scaling of the sensitivity $\Delta\delta \sim t^{-1}$ with the oscillation time $t$. The RI does not benefit from time scaling for the typical experimental times. However, the phase estimation with the RI has two important advantages with respect to those proposals. First, the perturbing potential is deduced from the measurement of the population imbalance, not from the interference pattern of an atomic cloud released from the optical lattice [29,30]. Counting atoms in dilute samples by making use of resonant light beams is a promising technique [5,6] and is expected to reach a very high signal-to-noise ratio. Moreover, differently from the Bloch oscillation proposal, the sensitivity of the RI can be quantum enhanced by the use of proper particle-entangled states. As shown, the phase sensitivity $\Delta\delta \sim N^{-\beta}$ scales at the shot noise $\beta = 1/2$ for the CSS and can be further increased to $1/2 < \beta \leqslant 1$ for spin squeezed states.

The MZI, in contrast, has two important advantages with respect to the RI: the wells can be separated to be far apart and the sensitivity scales with the inverse of time. Here we compare the sensitivity of the RI and the MZI fed with the CSS. When $\frac{\delta^2}{E_J^2} \ll 1$, the difference between the sensitivity of the MZI and that of the RI grows in time [33]. The smallest difference is $\Delta\delta = \frac{\pi}{2}\Delta\delta_{\mathrm{MZI}}$, obtained at the the first optimal time (60 ms for $\delta/\hbar = 4.4\ \mathrm{s}^{-1}$ and $E_J/\hbar = 52.3\ \mathrm{s}^{-1}$). On the contrary, the implementation of the MZI, which is composed of two balanced beam splitters, requires careful manipulation of the double-well potential [4–6]. As a key advantage, the realization of the RI does not require any coherent splitting or recombination of the atomic cloud and might therefore be easier to implement than the MZI.

## VII. CONCLUSIONS

We have shown that a degenerate bosonic or fermionic gas in a double-well potential can constitute a sensitive device for measuring short-range interactions, such as the Casimir-Polder force. We have demonstrated that, when the RI is fed with a classical spin coherent state with a moderate number of atoms, the Casimir-Polder force can be measured with a precision sufficient to distinguish between its thermal and its quantum, zero-temperature regime. Our predictions include possible sources of noise, as imperfect detection and residual atomic interaction, and optimization of the population imbalance measurement.

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